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The social discount rate: some implications of the budget-constrained opportunity cost approach

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THESIS

THE SOCIAL DISCOUNT RATE:
SOME IMPLICATIONS OF THE BUDGET-
CONSTRAINED OPPORTUNITY COST APPROACH

by

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The social discount rate is an important issue in the cost-benefit analysis for selecting public projects. However there has been no general consensus as to the appropriate value of the social rate of discount for public investment. In 1987, Quirk and Terasawa proposed using the opportunity cost rate of return as an alternative approach to the choosing of the social discount rate in a fixed-budget scenario. Essentially, the appropriate value of the government rate of discount is the highest rate of return available from the portfolio of the unfunded government projects.

In this study, the characteristics of the discount rates is explored in the context of choosing an efficient portfolio of government projects under fixed-budget condition. The costs and benefits of the projects are treated as variables and are to be endogenously determined by optimizing the overall discounted benefits. It is assumed that the costs and benefits of projects are known and continuous functions of force size, unit system maintenance, and operational support.

A mathematical model is used to represent the relationship between the benefit and cost of various projects. The Karesh-Kuhn-Tucker (KKT) convexity conditions are assumed for these so-called diminishing-return projects. In addition, two-year constant-returns-to-scale projects with fixed rate of return are introduced as reference projects such that their rates of return can be used directly as the discount rates under the concept of the opportunity cost rate of return.

The discounted present values (DPV) of the net benefits of both the optimal and non-optimal protfolios are found to be in agreement with those expected under the concept of the opportunity cost rate of return.

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THE SOCIAL DISCOUNT RATE: SOME IMPLICATIONS OF THE BUDGET-CONSTRAINED OPPORTUNITY COST APPROACH

by

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The discounted present values (DPV) of the net benefits of both the optimal and non-optimal portfolios are found to be in agreement with those expected under the concept of the opportunity cost rate of return.

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THESIS DISCLAIMER

The reader is cautioned that computer program developed in this research may not has been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the program is free of computational and logic errors, it cannot be considered validated. Any application of this program without additional verification is at the risk of the user.

I. INTRODUCTION

A. BACKGROUND

The concept and importance of the discount rate in the case where benefits and costs are spread over time are well recognized and understood. However there has been no general consensus as to the appropriate value of the social rate of discount for public investment. This controversy persists today.

The main approaches taken so far are the second-best approach and the opportunity cost approach. When government investment spending is at its optimum level and the social rate of time preference is known, both approaches provide similar results for the social rate of discount. However, the main difficulty has been the estimation of the social rate of time preference. Often the consumer rate of interest is incorrectly used in place of the social rate of time preference.

Instead of searching for the optimal level of government spending, Quirk and Terasawa (QT) proposed using the opportunity cost rate of return as an alternative approach to the choosing of the social discount rate in a fixed-budget scenario. As proposed, the appropriate value of the government rate of discount is the highest rate of return available from the portfolio of unfunded government projects.

B. PURPOSE OF STUDY

The purpose of this study is to explore the characteristics of social discount rate in the context of choosing an efficient portfolio of government projects under a

fixed-budget condition. The costs and benefits of the projects are treated as variables and are to be endogenously determined by optimizing the overall discounted benefits. Costs and benefits are endogenous because funding decisions can be revisited each year not only for initial investment costs for new projects but also for maintenance and support for old projects. It is assumed that the cost and benefit (input and output) of projects are known and continuous functions of force size (number of systems), unit system maintenance and operational support.

C. LITERATURE SURVEY

The social discount rate is an important issue in cost-benefit analysis for selecting public projects. In 1972 OMB Circular A-94 specified that a rate of 10% be adopted for all U.S. federal agencies and departments. Since then some progress on this issue has been made. But there is still lack of consensus on the appropriate choice of the social discount rate. Randolph Lyon [8] recently reviewed the practices in discounting public investments by the major U.S. government organizations and found many inconsistencies. A good account of the important developments pertaining to the social discount rate can be found in Lind [6] and QT [11].

In the so-called second-best approach, much of the work on the social discount rate has been concentrated on choosing a rate that properly adjusts for the market distortions caused by the corporate income tax. Usher [14] showed that the social discount rate lies between the consumer rate of interest and the pre-tax rate of return on corporate investment.

Another approach is one of opportunity cost. Using this approach, Ramsey [10] showed that the social rate of discount should be a weighted average of the consumer rate of interest and the pre-tax rate of return on corporate investment. The weight is a function of the fraction of resources drawn from consumption versus investment.

Both Usher and Ramsey derived their results based on a two-period model. By doing so, they avoided the issue of reinvestment of the proceeds of a project. To take this secondary effect into account, the shadow price of capital problem was first expounded by Marglin [9] and generalized by Bradford [1].

The idea is to convert all costs and benefits to consumption equivalents and then to apply a single rate of discount to the benefit and cost streams. The shadow price of capital is defined as the present value of the stream of consumption benefits associated with $1 of private investment discounted at the social rate of time preference. Drawing a distinction between the values of the shares of costs from consumption and investment, the present value of a public program is given as follows [S-39, 8]:

$$\sum_{t=0}^{T} \frac{X_t - [(1-c)\mu + c] C_t}{(1+i)^t}$$

where X_t is the benefits consumed at time t

C_t is the cost at time t

c is the proportion of consumption

μ is the shadow price of capital for which the rate of return equals the corresponding opportunity cost of capital

i is the rate of time preference

Lyon [8] showed that the shadow price of capital, μ, is highly sensitive to the opportunity cost of capital, time preference and reinvestment rate, whose values are uncertain by nature. Commonly, the consumer's rate of time preference is used in place of the social rate of time preference. This simplification is unacceptable, because it is true only for a single infinite-life consumer. Also, difficulties arise in choosing a social discount rate in the case of intergenerational equity [S-20, 7], [P.22, 11].

The second-best approach and the opportunity cost approach provide similar results for the social rate of discount if government investment spending is at an optimum and if the social rate of time preference equals the consumer rate of interest. However, the weights for the consumer rate of interest and the pre-tax rate of return on corporate investment differ for the two approaches. Given that the social rate of time preference is known, the second-best approach is the correct way to choose both the level of government investment and then the social rate of discount [P. 27, 11].

Lind [7], in his recent assessment, observed that international capital markets are becoming more integrated and the U.S. economy more open. He argued that this trend has a significant effect on the search for the discount rate. He suggested that many conclusions based on models of a closed economy where total government and private saving must equal private investment have to be modified significantly.

D. CONCEPT OF OPPORTUNITY COST RATE OF RETURN IN A FIXED BUDGET SCENARIO

In 1987, QT [11] proposed an alternative approach, the third-best, to the choosing of the social discount rate. As proposed, the appropriate value of the government rate of discount in a fixed-budget scenario should be the opportunity cost rate of return. This is the highest rate of return available from the portfolio of the unfunded government projects.

Firstly, it is not uncommon to find in practice the level of government expenditure being fixed by some political process with economic efficiency as one of many decision factors. The selection of projects has to be made within such budget constraints. The discount rate proposed for use by QT is the opportunity cost rate of return; this is the highest rate of return available from the set of unfunded projects. With this value of the discount rate, projects are funded only if the discounted present value (DPV) of net benefits from a project is positive. On the other hand, the unfunded projects shall have either zero or negative discounted present values.

The approach of opportunity cost rate of return is based purely on efficiency grounds. The social rate of time preference as needed in the other approaches is not required here. The intergenerational equity issue must be addressed separately. More importantly, the discount rate chosen acts as a filter against manipulation of estimates [P.41, 11], [P.21, 12].

One example is used to illustrate the application of the concept and the computation of DPVs in QT. The net benefits (benefits minus costs) for the available

projects over the period considered are predetermined separately. That is, the level of costs and benefits are decided in their respective project planning process. No communication between various project planners or adjustments to the relative allocation of fund is attempted. Under a given budget limit in each year, the highest rate of return of the unfunded projects is the choice of discount rate.

The example used for illustration in QT is reproduced in Table 1.1. A portfolio of four projects are available for investment. Using the opportunity cost rate of return approach, project B is funded. With project B funded, the portfolio of unfunded projects are A, C and D. In this case, the highest rates of return of the unfunded projects are 200% and 100% for period 0 and period 1 respectively. The opportunity cost rate of discount at t = 1 is the 100% of project D because it is the only rate of return available. Using the opportunity cost rates of return as discount rates, the discounted present values (DPV) of net benefits under project B is +$0.50, the DPV of net benefits of project A or D is 0, and DPV of net benefits of project C is -$0.33. It is observed that under the opportunity cost rate of return, project B is funded because it has a positive DPV of net benefits and it exhausts the investment funds available.

Table 1.1 Payoff of projects

Project	Payoff t = 0	Payoff t = 1	Payoff t = 2	Discounted Present Value (DPV)
A	-$1	$3	0	$0
B	-$1	0	$9	$0.50
C	-$1	$2	0	-$0.33
D	0	-$1	$2	$0
Discount Rate	200%	100%	-	

E. MOTIVATION OF THIS STUDY

The above example as illustrated in QT is a case where the projects are of constant returns to scale and their rates of return are fixed. The implication is that there is no consultation nor communication among the project planning offices. All project planning is carried out in isolation and optimization achieved at best locally within the respective project planning process. The effects of local optimization is reflected in the varying rates of return among the projects.

In this study, we are interested in a slightly more general scenario where there are always existing projects and newly-proposed projects in any year. All existing projects need funding for operational support and maintenance (O&M) in order to realize their benefits. Given a fixed budget for each year, the level of funding for O&M has to be decided against the newly-proposed projects. In other words, to achieve optimum allocation of the given budget in each year, all existing projects are to compete with other newly-proposed projects. This scenario is always valid at some higher echelon in the government organization.

This scenario in turn assumes that perfect flow of information occurs among the planning of various projects in a portfolio. Adjustment and redistribution is made in the allocation of monetary resources among the efficient projects such that the sum of all discounted outputs is maximized. Global optimization is the result of perfect information sharing. In this instance, a unique rate of return for each year is shared by all the selected projects.

Therefore, we plan to explore the characteristics of using the opportunity cost of rate of return as the discount rate in the context of choosing an efficient portfolio of government projects, whose costs and benefits are to be endogenously determined by optimizing the overall discounted benefits. Specifically we are interested in the discounted present values (DPV) of the net benefits of both the funded and unfunded projects.

For the purpose of this study, a mathematical model is used to represent the relations between the benefit and cost of each project. This model is described in the following chapter.

F. SCENARIO

1. A PORTFOLIO OF DEFENSE PROJECTS

All defense projects in the portfolio serve common ultimate goal(s), i.e., the benefits of one project can be replaced by others. Two types of projects are considered:

- diminishing return projects (DRP) with respect to the O&M funds allocated, and

- reference projects of constant returns to scale.

a. DIMINISHING-RETURN PROJECT (DRP)

Generally, there are two distinct periods considered for each project, i.e., initiation and operations. Initiation refers to the period when a project undergoes development and production. During this period, initial investment is needed to acquire the number of systems, initial spares, facilities, and basic training at the initiation of project. Obviously there is no benefit derived during this period.

After initiation, the project enters the operations period until retirement or upgrade. During this period, operational support and maintenance costs are incurred for the upkeep of systems and operational readiness on a yearly basis. Benefit or output is produced throughout the period. The level of output depends on force size and the combination of both unit maintenance and operational support.

The term diminishing return projects (DRP) refers to the ones where the rate of return (benefit) is diminishing with respect to the O&M fund allocated during the period of operations. In other words, the marginal rate of return decreases with an increase in the O&M cost.

b. REFERENCE PROJECT: CONSTANT RETURNS TO SCALE

In addition to the diminishing-return projects, constant-returns-to-scale projects are introduced as reference projects. The benefit of the reference project is directly proportional to the initial funds allocated. In this instance, the reference project has a two-year system life, with investment cost incurred in the year of

initiation and a return is produced at a known fixed rate in the following year. The rate of return of the reference projects is set so that it will serve as the highest rate of return of the unfunded projects.

Under the concept of opportunity cost rate of return, the fixed rates of return of these reference projects could be used appropriately as the discount rates in maximizing the overall benefits. In essence, the reference projects allow the discount rates to be treated as "exogenous", thus simplifying the computation of DPVs for the funded and unfunded projects while optimizing the overall discounted output of the selected projects.

2. BENEFITS OF DEFENSE PROJECTS

Benefits of defense projects are much more difficult to quantify by their very nature. One possible quantification is the measure of force effectiveness (MOFE). MOFE attempts to measure the performance of the force in a specific mission, taking into considerations the command and control, doctrine and tactics, force capabilities, etc. In addition, MOFE reflects the effectiveness of varying military capabilities and the marginal effects of changes in force levels. The unit of MOFE varies according to scenario. In this study, without delving into details, we assume that the unit of MOFE is convertible to an equivalent dollar value. This assumption is necessary to make the costs and benefits directly comparable.

3. FIXED BUDGET

In the short run, it is fair to assume that the budget stream over the near future is fixed. Under this budget constraint, the more efficient projects are to

be identified. The overall objective of defense planning then is to derive maximum benefits within the such constraints.

II. THE MODEL

A. INTRODUCTION

A mathematical model that represents the relationship between the cost and benefit of a project is developed to facilitate the study of the discount rate and its related issues. Using only essential factors, this model is intentionally kept simple, yet it is able to capture some of the main features of the real world. The main factors considered are the number of systems, system unit maintenance and its residual effect, and system unit operational support.

B. ASSUMPTIONS

The following assumptions are made:

1. Input and output functions are nonlinear and continuous. For the purpose of this study, the input and output of each project are assumed to be non-linear functions of force size (i.e., number of systems), and system unit maintenance and operational support. Continuous functions imply a continuous solution space, within which the solution of optimum resource allocation lies. In addition, the continuity assumption facilitates the search for the optimum solution using numerical methods based on Karesh-Kuhn-Tucker (KKT) conditions.

2. For the diminishing-return projects, the original benefit (output) is monotonically decreasing with the system unit maintenance and support, and in fact

we assume the benefit to be a concave function. The marginal cost (input) is monotonically increasing with the system unit maintenance and support, or a convex function. Mathematically maximizing the concave output function subject to convex inequality constraints is a sufficient optimality condition for KKT (or KKT convexity conditions) [p.139, 2]. In general, the optimum solution found using KKT conditions is local, and there is no assurance of a global optimization. The KKT convexity condition used here is sufficient to ensure a global optimization.

3. The force size, or number of system, is assumed to be scaleable and divisible in this study. That is, the number of systems is a continuous variable instead of a discrete variable. This assumption is particularly acceptable for large force sizes where the effect of divisibility becomes negligible.

C. THE MODEL

The mathematical expression of the objective function, input and output functions, and budget constraint are described in this section. As mentioned earlier, for each project, two distinct periods (without overlap) in the entire system life are considered, i.e., initiation and operations. The year when a project begins is the year of project initiation. During this initial period, the systems undergo development and production. For simplicity, the cost of systems incurred during project initiation is lumped into a one-year period so as to be consistent with the following operating years. Since the system is under development and production, there is no benefit produced during this period.

After the period of initiation, the systems proceed to a period of operations until system retirement or upgrade. During the entire operating period, maintenance and operational support costs are incurred on a yearly basis. Naturally benefit or output is produced throughout the system operations. The level of output would depend on the combination of both the unit system maintenance and operational support. Maintenance includes both the scheduled and unscheduled maintenance of the systems. The operational supports entail training of operators, exercises, transportation of systems, operational deployment, etc.

All projects could begin or end in any year within the period considered. The subscripts i and t represent the project and year, respectively.

1. Objective Function

The objective is to maximize the sum of the discounted benefits of all selected projects over the period considered. The discount rates used in the objective function are those generated by the opportunity cost rate of return concept. It is assumed that benefit is derived from a project throughout its entire life except in the year of initiation. Under the concept of opportunity cost rate of return as the discount rate, the discount rate is the highest rate of return available from the set of unfunded projects in each year. The objective function is given as follows:

$$Max \quad \sum_{t=1}^{T} \sum_{i=1}^{I} D_t * X_{it} \; ... \; (1),$$

where X_{it} is the output of project i in year t

D_t is the discount factor for year t and it is given

by following expression,

$$D_t = \frac{1}{\prod_{\tau=1}^{t} (1+dr_\tau)}$$

where dr_τ is the discount rate for year τ

2. Fixed Budget Level

As explained earlier, the budget level for each year, B_t, is predetermined. Thus the sum of all budget allocations for the selected projects must not exceed this budget limit. There are three types of cost factors considered for each of the selected projects:

a. cost of initial investment, IC_i

b. cost of maintenance, CM_{it}

c. cost of operational support, CS_{it}

The cost of initial investment is a function of unit system cost, K_i, and force size or number of systems, N_i. Initial investment is incurred in the year when the project begins. Those projects that are not funded shall have zero initial investment with no systems acquired. On the other hand, the costs of maintenance and operational support are incurred in the years following the year of initiation until the end of system life. The cost functions of maintenance and operational support are described in later paragraphs. The budget constraint is given as follows:

$$\sum_{i=1}^{I} [(IC_i)_{q_i=t} + (CM_{it} + CS_{it})_{q_i<t}] \leq B_t, \quad \forall t \ldots (2a)$$

or

$$\sum_{i=1}^{I} [(K_i * N_i)_{q_i=t} + (CM_{it} + CS_{it})_{q_i<t}] \leq B_t, \quad \forall t \ldots (2b)$$

where B_t is the budget limit in year t

K_i is the unit system cost of project i

N_i is the number of systems of project i

q_i is the starting year for project i

3. Benefit (Output) Function

Benefit or output is produced by each project when the system is in operation. The output of the project in each year is a function of force size, and the unit system maintenance and operational support, i.e.,

$$X_{it} = N_i * (u_i * M_{it}^{a_i} * S_{it}^{b_i}), \quad \forall i, t \ldots (3)$$

where X_{it} is the output of project i in year t

u_i is a conversion factor from benefit to dollar value

N_i is the number of systems acquired with project i

M_{it} is the effective unit system maintenance level for project i in year t

S_{it} is the unit system support level of project i in year t

a_i is the exponent of M_{it}

b_i is the exponent of S_{it}

The exponents a_i and b_i are the elasticities that shape the effects of unit system maintenance and support on the level of output. They are the measures of the percentage change in the level of output in response to a one percent change in its unit system maintenance level and unit system support level, respectively, i.e.,

$$a_i = \frac{(\frac{\Delta X_{it}}{X_{it}})}{(\frac{\Delta M_{it}}{M_{it}})} \quad and \quad b_i = \frac{(\frac{\Delta X_{it}}{X_{it}})}{(\frac{\Delta S_{it}}{S_{it}})}$$

In this study, the exponents or elasticities for each project is held constant throughout the system life. The sum of a_i and b_i is less than one, which is also homothetic, satisfies the KKT convexity conditions. Additionally, the output is directly proportional to the number of systems.

4. Residual Characteristics of Maintenance

The idea of residual maintenance is perceived from the fact that the effects of maintenance performed in the previous year does not usually disappear suddenly. Instead, maintenance is assumed to be decaying at some constant rate, called the residual index, d, of value between 0 and 1. A zero value for d means no residual effects, while a d of one signifies that the maintenance effects are perpetual. In general, the residual index represents a percentage of maintenance level that is carried forward to the following year. This condition also serves as a link between the

years of operation of the projects. Hence, the maintenance function is formulated as:

$$M_{it} = M_{i,t-1} * d_i + m_{it}, \quad \forall\ i,t \ \ldots\ (4)$$

where m_{it} is the flow of unit system maintenance actually provided to project i in year t

M_{it} is the stock of unit system maintenance representing the sum of m_{it} and the residual effects of maintenance from previous year, i.e., $M_{i,t-1}$

d_i is the residual index of maintenance for project i

5. Cost Functions

There are two kinds of costs incurred during operations: maintenance cost and operational support cost. The KKT convexity condition requires the marginal cost to be monotonically increasing with respect to the levels of unit system maintenance and operational support, while the marginal output be monotonically decreasing.

a. Maintenance Cost Function

Maintenance cost of each project in each year is a function of force size and level of unit system maintenance. The cost is assumed to be directly proportional to the number of systems. The maintenance cost of each project in each year (CM_{it}) is given as follows:

$$CM_{it} = N_i * (v_i * m_{it}^{\alpha_i}), \quad \forall\ i,t \ \ldots\ (5)$$

where v_i is a conversion factor from level of maintenance

to dollars

N_i is the number of systems acquired with project i

m_{it} is the flow of unit system maintenance

actually provided to project i in year t.

α_i is the exponent of m_{it}

The exponent α_i can be interpreted as the elasticity, i.e., the measure of the percentage change in the cost of maintenance in response to a one percent change in its level of system unit maintenance. Under the KKT convexity condition, the elasticity, α_i, has to be greater than one. In addition, it assumes a constant value throughout the system life.

$$\alpha_i = \frac{\left(\frac{\Delta CM_{it}}{CM_{it}}\right)}{\left(\frac{\Delta m_{it}}{m_{it}}\right)}$$

b. Operational Support Cost Function

Operational support cost of each project in each year is a function of force size and level of unit system operational support. Again the cost is assumed to be directly proportional to the number of systems. The operational support cost of each project in each year (CS_{it}) is given as follows:

$$CS_{it} = N_i * (w_i * S_{it}^{\beta_i}), \quad \forall\, i,t \ \ldots \ (6)$$

where w_i is a conversion factor from level of support to dollar

value

N_i is the number of systems acquired with project i

S_{it} is the level of support provided to project i in year t

β_i is the exponent of m_{it}

Like α_i, the economic interpretation of the exponent β_i is the elasticity, i.e., the measure of the percentage change in the cost of operational support in response to a one percent change in its unit system support level. Similarly, under the KKT convexity condition, the elasticity, β_i, has to be greater than one. In addition, β_i assumes a constant value throughout the system life.

$$\beta_i = \frac{\left(\frac{\Delta CM_{it}}{CM_{it}}\right)}{\left(\frac{\Delta m_{it}}{m_{it}}\right)}$$

D. SIMPLIFIED ANALYTICAL FRAMEWORK

In this section, a simplified analytical framework, where all diminishing-return projects begin in year 1, is provided. As presented in the earlier section, the sum of discounted benefits of all funded projects over the years considered are to be maximized and expenditures are subject to the budget constraint in each year. Besides the benefit and cost functions, the lingering effects of maintenance are known. The analytical approach to solving this formulation is that of the Lagrange method, i.e.,

$$L = \sum_{t=1}^{T} \sum_{i=1}^{I} D_t * (N_i * u_i * M_{it}^{b_i} * S_{it}^{c_i}) + \mu_1 * (B_1 - \sum_{i=1}^{I} N_i * K_i)$$

$$+ \sum_{t=2}^{T} \mu_t * (B_t - \sum_{i=1}^{I} N_i * (v_i * m_{it}^{\alpha_i} + w_i * S_{it}^{\beta_i})) \quad \ldots \quad (7)$$

where

$$D_t = \frac{1}{\prod_{\tau=1}^{t} (1+dr_\tau)},$$

and μ_1 and μ_t are the Lagrange multipliers for the

initial year and the subsequent years respectively.

The Lagrange multipliers, μ_1 and μ_t are associated with budget constraints for the initial and succeeding years. The Lagrange multiplier of any year is also called the shadow price of that budget constraint and is defined as the change in the overall discounted outputs with respect to the change in budget level in that year for this problem, i.e.,

$$\mu_t = \frac{\partial X}{\partial B_t}$$

where X is the sum of overall discounted output

B_t is the budget constraint for year t.

As it is, equation (7) cannot be solved directly if the discount factor, D_t, is treated as a variable. In this instance, the discount factor is the inverse of a compound

factor of discount rates, each being an unknown variable. In order to use the existing method of Lagrange, in this study, the discount factor has to be a given parameter (this point is further explained later in Section F of this chapter). Now, differentiating equation (7) with respect to m_{it} and S_{it}, and setting them to zero, we have the following:

$$m_{is}: \quad \sum_{t=s}^{T} D_t * (N_i * u_i * a_i * M_{it}^{a_i - 1} * d_i^{t-s} * S_{it}^{b_i})$$

$$- \mu_s * N_i * v_i * \alpha_i * m_{is}^{\alpha_i - 1} = 0 \ , \ \forall\, i \ldots \ (8)$$

$$S_{it}: \quad D_t * (N_i * u_i * M_{it}^{a_i} * b_i * S_{it}^{b_i - 1})$$

$$- \mu_t * N_i * w_i * \beta_i * S_{it}^{\beta_i - 1} = 0 \ , \ \forall i, t \ \ldots (9)$$

However, the above set of equations is still too complicated to be solved analytically. One way is to allow some of the input parameters to assume some convenient values. Such simplification has been used to verify the results of the computer implementation (which is described in Chapter III).

E. OUTPUT PARAMETERS

There are two main output parameters to be computed from the above model. They are the shadow prices and the discounted present value (DPV) of the net benefits:

1. Shadow Prices

As defined earlier, the shadow price in this problem represents the change in the overall discounted benefits of an additional unit of budget allocation. It is a function of budget limit in each year. Under the condition of global optimization, all selected projects in the same year share the same shadow price.

2. Discounted Present Value (DPV) of the Net Benefits

The discounted present value (DPV) of the net benefits of each project is the sum of all discounted net benefits of the entire system life, i.e.,

$$DPV_i = -IC_i + \sum_{t=2}^{T} D_t * (X_{it} - C_{it}) \quad \ldots \quad (10)$$

where

$$D_t = \frac{1}{\prod_{\tau=1}^{t} (1+dr_\tau)}$$

$IC_i = K_i * N_i$

X_{it} is the output of project i in year t

dr_τ is the discount rate for year τ

$C_{it} = CM_{it} + CS_{it}$

F. DILEMMAS AND THEIR REMEDIES

Three dilemmas are encountered in the process of implementing the above initial formulation. They are summarized as follows:

1. Before solving the objective function subject to the budget constraints, we have no knowledge of which projects from the portfolio would be selected for funding. In other words, the set of unfunded projects is not known. This information is essential in identifying the rate of discount.

2. Values of appropriate discount rates are not directly available from the above formulation. This is because the relation between shadow price and rate of return is necessary yet it is not available.

3. Related to the first problem, since unfunded projects are excluded from funding, the shadow prices of these unfunded projects will not be available from the solution for the initial formulation.

To get around the problems, some two-year reference projects are introduced for convenience. The characteristic of each reference project is such that investment cost is incurred in the year of initiation and return is produced at a known fixed rate in the following year. The fixed rate of this reference project is set at an appropriate level so that it can be treated as the highest rate of return of some unfunded projects at the margin. Therefore it can be used directly as the discount rate in the objective function under the concept of opportunity cost of rate of return. With these rates from the reference projects, the other diminishing-return projects in the portfolio that have lower rates of return will be excluded from funding. The discounted present value

(DPV) of the net benefits of the funded projects will be observed in the process of optimizing the overall discounted output.

The output of the reference project is directly proportional to the investment cost in the previous year, i.e.,

$$R_t = (1+r_t)*RC_{t-1} \quad ...(11)$$

where R_t is the output of the reference project in year t

r_t is the rate of return of the reference project in year t

RC_{t-1} is the investment cost of reference project in the previous year

To find out the DPVs of the unfunded projects or non-optimal set of projects, a two-step procedure is adopted. In the first optimization process, the optimal set of (efficient) projects is identified. Then using any non-optimal portfolio, by removing any of the efficient project(s), under same budget constraint and known rates of discount, the DPVs for the non-optimal set of projects are to be calculated in the second optimization process. The modified version the entire formulation can be summarized as follows:

$$Max \quad \sum_{t=1}^{T} \left[\sum_{i=1}^{I} D_t*X_{it} + D_t*R_t \right]$$

where

$$D_t = \frac{1}{\prod_{\tau=1}^{t} (1+dr_\tau)}$$

$$dr_t = r_{t-1} , \quad \forall \ t$$

(discount rate, dr_t is the rate of return of

the reference project invested in year (t-1))

Subject to:

$$\sum_{i=1}^{I} [\ (K_i * N_i)_{s_i = t} + (CM_{it} + CS_{it})_{s_t, t} \ ] + RC_t \leq B_t, \quad \forall \ t$$

where

$$X_{it} = u_i * N_i^{a_i} * M_{it}^{b_i} * S_{it}^{c_i}, \quad \forall \ i, t$$

$$M_{it} = M_{i,t-1} * d + m_{it}, \quad \forall \ i, t$$

$$R_t = (1+r_t) * RC_{t-1}, \quad \forall \ t$$

$$CM_{it} = v_i * N_i * m_{it}^{\alpha_i}, \quad \forall \ i, t$$

$$CS_{it} = w_i * N_i * S_{it}^{\beta_i}, \quad \forall \ i, t$$

Decision variables:

$$X_{it}, \ M_{it}, \ m_{it}, \ S_{it}, \ N_i, \ R_{jt}, \ C_{jt} \quad \forall \ i, j, t$$

III. IMPLEMENTATION USING GAMS

A. INTRODUCTION

Instead of using an analytical approach to solve the modified formulation, numerical solutions are computed based on the Karesh-Kuhn-Tucker (KKT) conditions. The formulation is implemented in GAMS, a software package for solving optimization problems.

B. NONLINEAR PROGRAMMING (NLP)

In nonlinear programming, the objective function provides the desired direction for improvement and the set of constraints defines the feasible space in which the optimum solution lies. The search for the optimum solution begins with an arbitrary point in the space, then proceeding toward the "improving direction", at each step satisfying the Karesh-Kuhn-Tucker (KKT) conditions [2]. The search continues until a point where there is no more "improving direction" within a specified tolerance.

C. GAMS

GAMS (the acronym stands for General Algebraic Modeling System) [3], a software package, is chosen to implement the NLP model. Specifically, a solver called MINOS5 is used. Using algebraic modeling concepts, GAMS makes the construction and solution of large and complex mathematical programming models more straightforward and comprehensible. The GAMS listing are given in Appendix A.

Due to the nature of the search, the appropriate KKT convexity condition is assumed for the benefit and cost functions in order to ensure a global optimum solution. Two output parameters are generated by the GAMS program: shadow prices and the discounted present values (DPV) of the net benefits.

D. ADVANTAGES OF GAMS

The advantages of GAMS implementations are as follows:

a. fast execution time (using a mainframe computer)

b. allows change of input parameters with ease, e.g., different starting and ending years of project. (In the case of an analytical approach, such changes in input entail a new set of derivations.)

IV. NUMERICAL ANALYSIS USING THE MODEL

A. INTRODUCTION

In this illustration, a portfolio of three diminishing-return projects and three reference projects are considered over a period of three years. All diminishing-return projects begin in year 1 and end in year 3. The reference projects are constant returns to scale and their rates of return are set so that they will constitute the marginal project in each period. The three reference projects start in year 1, year 2 and year 3, respectively. Note that the only project that produces in year 4 is reference project 3.

By construction, we are able to use the rates of return of the reference project as the discount rate in the objective function, i.e., the sum of the discounted output of the selected projects. The aim of this analysis is to observe the resulting discounted present values (DPV) of the net benefits of the optimal and non-optimal set of projects, and to find out if they are congruent with the expectation of the concept of opportunity cost rate of return.

B. INPUT PARAMETERS

As mentioned, all diminishing-return projects start in year 1 and end in year 3. Year 1 corresponds to the year of initiation, while year 2 and 3 are for operations and thus output is produced. At this stage, we assume a residual maintenance of 10% for all three diminishing-return projects. The input parameters for the three

diminishing-return projects are tabulated in Table 4.1.

The rates of return for reference projects 1, 2 and 3 are 0.536, 0.3 and 0.3 respectively as shown in Table 4.2. The rate of return for reference project 1 is set at 0.536 because it would not be selected at any lower rate. Under the concept of opportunity cost of rate of return, all reference projects are to be the marginal projects, whose rates of return are the highest among unfunded projects for the particular year, before their rates of return could be used as the discount rates. At a rate of return of 0.536, reference project 1 becomes the marginal project (refer to Appendix B for more details). Finally the budget limits for this analysis are 15, 20 and 20 units for year 1, 2 and 3 respectively, as shown in Table 4.3.

Table 4.1 Input Parameters for Projects

	Exponents in Benefit Function		Unit System Price	Exponents in Cost Function		Conversion Factors			Residual Maintenance
Project	a	b	K	α	β	u	v	w	d
1	0.6	0.4	0.14	1.5	2.0	0.56	0.21	0.66	.1
2	0.5	0.4	0.28	1.8	2.0	0.70	0.35	0.20	.1
3	0.2	0.8	0.30	2.0	1.5	0.70	0.45	0.40	.1

Table 4.2 Input Parameters for Reference Projects

Reference Project	Rate of Return (G)
1	0.536
2	0.3
3	0.3

Table 4.3 Budget Limits

Year	Budget
1 2 3	15 20 20

C. STEP 1 - DPV OF THE OPTIMAL PORTFOLIO

As mentioned in Section F of Chapter II, at the first step of the two-step procedure, it is necessary to identify the optimal set of projects and the unfunded projects while optimizing the total discounted output within the budget constraints. As shown in Table 4.4, the projects selected are project 2 and reference projects 2 and 3. The set of unfunded projects are projects 1 and 3, and reference project 1. The various costs incurred in funding these projects are tabulated in Table 4.5.

Table 4.4 Undiscounted Output of the Funded Projects

Funded Set	Year 2	Year 3	Year 4	Total
Project 2	24.9	24.9	-	48.9
Ref. Proj. 2	-	9.9		9.9
Ref. Proj. 3	-		9.9	9.9
Total	24.9	34.9	9.9	71.1

Table 4.5 Cost Measure of the Funded Projects

Funded Set	Year 1	Year 2	Year 3
Project 2 (Maint. & Sup.)	15.0 - -	12.4 7.4 5.0	11.3 6.3 5.0
Ref. Proj. 2	-	7.6	-
Ref. Proj. 3	-	-	8.7
Total	15.0	20.0	20.0

Table 4.6 shows the DPVs of the selected project. For the given discount rates, project 2 has a positive DPV of slightly above zero (i.e., 0.004), while reference projects 2 and 3 have values of zero.

Table 4.6 Discounted Present Value (DPV) of the Net Benefits for the Funded Projects

Funded Set	Year 1	Year 2	Year 3	Year 4	DPV
Project 2	-15.0	12.5	13.8	-	0.004
Ref. Proj. 2	-	-7.6	9.9	-	0
Ref. Proj. 3	-	-	-8.7	11.4	0
Total	-15.0	16.9	8.6	11.4	0.004

Examination of the shadow prices provides some insight. When reference projects are selected, the shadow price for that particular year is compelled to take on the value of the corresponding rate of return. This is necessary so as to maintain a uniform shadow price for all selected projects in any year.

Table 4.7 Shadow Prices

	Year 1	Year 2	Year 3
Shadow Prices, $\mu(t)$	1.001	0.651	0.501
$\mu(min)$ *	1.00	0.651	0.501
Discount Rate	0	0.536	0.3
Discount Factor	1	1.536	1.997

* Right-hand side of equation (14)

In general, when maximizing the sum of the discounted output, the relationship among shadow prices in any year, μ_t, and the rates of return of reference projects and the choice of discount rates are given as follows:

$$\mu_t \geq \frac{1+r_t}{\prod_{i=\tau}^{t+1} (1+dr_\tau)}, \quad \forall\, t \qquad \ldots(13)$$

where r_t is the rate of return of reference project for year t-1

dr_τ is the discount rate for year τ, and $dr_1 = 0$

The equality holds whenever the reference project is selected for funding. This is observed in Table 4.7 where reference projects 2 and 3, are selected in year 2 and 3 respectively (note the equality between μ_t and μ(min)). Otherwise the shadow price is strictly greater than the right hand side of equation (13). This can be observed in year 1 of this example as presented in Table 4.7. In this instance, the diminishing-return projects achieve slightly higher returns than the reference project 1.

In this illustration, since the rates of return of the reference projects are used as the discount rate for the following year (i.e., $dr_{t+1} = r_t$), the above equation is reduced as follows:

$$\mu_t \geq \frac{1}{\prod_{\tau=1}^{t} (1+dr_\tau)}, \quad \forall\, t \qquad \ldots(14)$$

D. STEP 2 - DPV OF THE NON-OPTIMAL PORTFOLIO

The funded projects have been identified in step 1, i.e., project 2 and reference projects 2 and 3. Now at step 2, three non-optimal portfolios are considered, in each case an efficient project is removed, as shown in Table 4.8. The DPV for each non-optimal portfolio is determined by optimizing the overall discounted output, under the same budget constraint and discount rates. The status of the projects in each non-optimal portfolio is shown Table 4.8. The DPV of portfolio 1 is zero because only the reference projects are selected, while the DPVs of the other two are -0.6 and -0.64 (detailed output are shown in Appendix C). These results of zero or negative DPV for the non-optimal portfolios are consistent with the expectation of the concept of opportunity cost rate of return.

Table 4.8 Summary of DPVs of the non-optimal portfolio

Non-Optimal Portfolio	1	2	3
Proj. 1 Proj. 2 Proj. 3	unfunded removed unfunded	unfunded funded unfunded	unfunded funded unfunded
Ref. Proj. 1 Ref. Proj. 2 Ref. Proj. 3	funded funded Funded	unfunded removed funded	unfunded funded removed
DPV	0	-0.60	-0.64

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The debate on the issue of the social discount rate will continue. This study does not claim or intend to address all the problems at once. Instead the concept of opportunity cost rate of return, as proposed by Quirk and Terasawa, is examined in a slightly more general scenario where the portfolio of government projects, whose costs and benefits are to be endogenously determined by optimizing the overall discounted benefits. The initial formulation given in this report is an attempt to provide an operational structure to approach this problem. However, it was found difficult to solve in the context of a GAMS program. The main reason is because the set of unfunded projects is not known before the initial formulation is solved. Yet the knowledge of the set of unfunded projects is essential in identifying the appropriate choice of the discount rates.

To study the nature of the funded and unfunded projects, reference projects of constant returns to scale are introduced. Their rates of return are fixed such that they can be used as the discount rates, thus making the discount rates constant in the objective function. The modified formulation is then directly solvable. A numerical solution based on the Karesh-Kuhn-Tucker (KKT) conditions is used to solve the formulation.

The resulting DPVs were found to be in agreement with those expected under the concept of opportunity cost rate of return. The DPV of the optimal portfolio is positive, while the DPVs of the non-optimal portfolios are either zero or negative.

The concept of opportunity cost rate of return as discount rate is a viable alternative in the selection of the social rate of discount. The attempt to illustrate the concept using more general cost and benefit functions in this study does not in any way affect the foundation of the concept. What this study revealed is still a significant verification of the concept of opportunity cost rate of return, and is undoubtedly encouraging.

B. RECOMMENDATIONS

Further research areas on the characteristics and the related issues pertaining to the opportunity cost rate of return as the social discount rate are suggested as follows:

- Since the set of unfunded projects has an effect on the choice of discount rate, the impacts of different rates of return of the reference projects on the discounted present values (DPV) of the net benefits of various projects, both funded and unfunded, could be further examined.

- The residual effect of maintenance is an issue which deserves further study. Such efforts could include exploration of alternative models and validation of this phenomenon using real data.

- In this study, the KKT convexity conditions are used to shape the relationship between cost and benefit of the diminishing-return projects. Other kinds of relationships could be considered in future studies.

APPENDIX A: PROGRAM LISTING

```
$TITLE STUDY OF DISCOUNT RATES
$OFFUPPER OFFSYMLIST OFFSYMXREF
OPTION LIMROW = 0, LIMCOL = 0, SOLPRINT = OFF;
OPTION OPTCA = 0.1;
$ONTEXT
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*              D I S C O U N T   R A T E                   *
*                  VERSION 11.3 NLP                        *
*                                                          *
*                  2.9.91 - NG K C                         *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

THIS IS AN GAMS IMPLEMENTATION OF A MATHEMATICAL MODEL THAT DEPICTS THE
RELATIONSHIP BETWEEN COSTS AND BENEFITS OF PROJECTS IN A PORTFOLIO. IT IS
DEVELOPED TO FACILITATE THE STUDY OF DISCOUNT RATE, USING THE OPPORTUNITY COST
CONCEPT OF RATE OF RETURN. THE MAIN FACTORS CONSIDERED ARE NUMBER OF SYSTEMS,
UNIT SYSTEM MAINTENANCE AND ITS RESIDUAL EFFECT, AND UNIT SYSTEM OPERATIONAL
SUPPORTS.

$OFFTEXT

 SETS  I   DRP PROJECTS       / PROJ1*PROJ3 /
       T   YEARS              / YEAR1*YEAR4 /
       K   DATA SET 1 (DRP) / B,C,A,P,ALPHA,BETA,U,V,W,D,H/
       L   CONSTRAINTS        / REQUIRE, BUDGET /
      I2   REF PROJECTS       / REF1*REF3 /
      KK   DATA SET 2 (REF) / PP,R,HH/   ;

 ALIAS (T,TT);
 ALIAS (I,II);

TABLE BEGIN(I,T) STATUS OF DIMINISHING-RETURN PROJECTS OVER THE YEARS
        YEAR1 YEAR2 YEAR3 YEAR4
PROJ1     1     0     0    -1
PROJ2     1     0     0    -1
PROJ3     1     0     0    -1
PROJ4    -1     1     0     0   ;

* 1: PERIOD OF INITIATION
* 0: PERIOD OF OPERATIONS
* -1: PROJECT INACTIVE

TABLE BB(I2,T) STATUS OF REFERENCE PROJECTS OVER THE YEARS
        YEAR1 YEAR2 YEAR3 YEAR4
REF1      1     0    -1    -1
```

```
REF2      -1     1     0    -1
REF3      -1    -1     1     0  ;

TABLE DATA(I,K) INPUT PARAMETERS FOR DIMINISHING-RETURN PROJECTS

         B   C  A   P   ALPHA BETA   U    V    W   D  H
 PROJ1  .6  .4  1  .15   1.5   2.0   .60 .20 .65 .1  1
 PROJ2  .5   1  1  .25   1.8   2.0   .72 .35 .20 .1  1
 PROJ3  .9  .8  1  .40   2.0   1.5   .70 .45 .40 .1  1

PARAMETER DR(T)  DISCOUNT RATES FOR YEAR T
     /YEAR1   0
      YEAR2   0.536
      YEAR3   0.3
      YEAR4   0.3 /;


TABLE DATA2(I2,KK) INPUT PARAMETERS FOR REFERENCE PROJECTS
             PP    R    HH
 REF1        1   1.536  1
 REF2        1   1.30   1
 REF3        1   1.30   1          ;

TABLE LIMIT(L,T) REQUIREMENT AND BUDGET CONSTRAINTS

          YEAR1   YEAR2   YEAR3   YEAR4
 REQUIRE    0      .0      .0      .0
 BUDGET   15.0    20.0    20.0    20.0   ;

* REQUIRE: OUTPUT (BENEFIT) REQUIREMENTS
* BUDGET: BUDGET LIMITS

PARAMETER DRA(T)  DISCOUNT FACTORS;
DRA("YEAR1" = 1 + DR("YEAR1");
LOOP(T, $(ORD(T) NE 1), DRA(T) = (1+DR(T))*DRA(T-1));
DISPLAY DRA;

 VARIABLES
    Z          OBJECTIVE VARIABLE;

 POSITIVE VARIABLES
    N(I)       NUMBER OF SYSTEMS
    E(T)       BUDGET DEFICIT
    M(I,T)     MAINTENANCE LEVEL
    MM(I,T)    CUMULATIVE MAINTENANCE LEVEL
    S(I,T)     OPERATIONAL SUPPORT LEVEL
    IC(I,T)    INITIAL COST OF EACH PROJECT
    CC(I2,T)   COST OF REFERENCE PROJECT  ;

 EQUATIONS
```

```
    OBJ            OBJECTIVE FUNCTION
    REQUIRE(T)     OUTPUT REQUIREMENTS
    COST(T)        BUDGET CONSTRAINTS
    PROJCOST(I,T)  COST OF PROJECT I IN YEAR T
    DECAY(I,T)     DECAY OF MAINTENANCE ;

 OBJ..         Z =E= SUM(T,
 SUM(I $((DATA(I,"H") EQ 1)AND(BEGIN(I,T) EQ 0)),DATA(I,"U")*
          (N(I)**DATA(I,"A"))*
 ((1+DR)**(1-ORD(T)))*(MM(I,T)**DATA(I,"B"))*(S(I,T)**DATA(I,"C")))
    - 50*E(T) ) + SUM((T,I2)
    $((DATA2(I2,"HH") EQ 1)AND(BB(I2,T) EQ 0)),
    ((1+DR)**(1-ORD(T)))*DATA2(I2,"R")*CC(I2,T-1) )  ;

 REQUIRE(T)..  SUM(I $((DATA(I,"H") EQ 1)AND(BEGIN(I,T) EQ 0)),
     DATA(I,"U")*(N(I)**DATA(I,"A"))*
   (MM(I,T)**DATA(I,"B"))*(S(I,T)**DATA(I,"C"))) +
  SUM(I2 $((DATA2(I2,"HH") EQ 1)AND(BB(I2,T) EQ 0)),
     DATA2(I2,"R")*CC(I2,T-1)) =G= LIMIT("REQUIRE",T);

 COST(T).. SUM(I, (DATA(I,"V")*N(I)*(M(I,T)**DATA(I,"ALPHA"))
     + DATA(I,"W")*N(I)*(S(I,T)**DATA(I,"BETA")))
       $((DATA(I,"H") EQ 1) AND (BEGIN(I,T) EQ 0))
     + (DATA(I,"P")*N(I))
       $((DATA(I,"H") EQ 1) AND (BEGIN(I,T) EQ 1))) +
   SUM(I2, CC(I2,T) $((DATA2(I2,"HH") EQ 1) AND (BB(I2,T) EQ 1)))
            =L= LIMIT("BUDGET",T) + E(T);

 DECAY(I,T) $((DATA(I,"H") EQ 1) AND (BEGIN(I,T) EQ 0))..
                MM(I,T) =E=
   MM(I,T-1)*DATA(I,"D") $(BEGIN(I,T-1) EQ 0) + M(I,T);

** BOUNDS OF VARIABLES **
E.FX(T) = 0;
N.LO(I) $(DATA(I,"H") EQ 1) = 0.0001;
N.L(I)  $(DATA(I,"H") EQ 1) = 1.0;
MM.LO(I,T) $(DATA(I,"H") EQ 1) = 0.001;
M.LO(I,T) $(DATA(I,"H") EQ 1) = 0.001;
S.LO(I,T) $(DATA(I,"H") EQ 1) = 0.001;
MM.L(I,T) $(DATA(I,"H") EQ 1) = 1;
M.L(I,T) $(DATA(I,"H") EQ 1) = 1;
S.L(I,T) $(DATA(I,"H") EQ 1) = 1;

 MODEL DISCOUNT /OBJ, REQUIRE, COST, DECAY/ ;
 SOLVE DISCOUNT USING NLP MAXIMIZING Z ;

*** OUTPUT OF RESULTS ***

DISPLAY DATA;
DISPLAY DR;
DISPLAY DATA2(I2,"R");
```

```
PARAMETER REPORT2(*,*) UNDISCOUNTED OUTPUT MEASURES;
REPORT2(I,T) $((DATA(I,"H") EQ 1) AND (BEGIN(I,T) EQ 0)) =
DATA(I,"U")*(N.L(I)**DATA(I,"A"))*
((MM.L(I,T)**DATA(I,"B"))*(S.L(I,T)**DATA(I,"C")));
REPORT2(I2,T) $((DATA2(I2,"HH") EQ 1) AND (BB(I2,T) EQ 0)) =
DATA2(I2,"R")*CC.L(I2,T-1);
REPORT2(I,"TOTAL") = SUM(T, REPORT2(I,T)) ;
REPORT2(I2,"TOTAL") = SUM(T, REPORT2(I2,T));
REPORT2("TOTAL",T) = SUM(I $(DATA(I,"H") EQ 1), REPORT2(I,T)) +
                     SUM(I2 $(DATA2(I2, "HH") EQ 1),
REPORT2(I2,T));

REPORT2("TOTAL","TOTAL") = SUM(T, REPORT2("TOTAL",T));
REPORT2("REQ'D",T) = LIMIT("REQUIRE",T);
REPORT2("REQ'D","TOTAL") = SUM(T, REPORT2("REQ'D",T));
REPORT2("EXCESS",T) = REPORT2("TOTAL",T) - REPORT2("REQ'D",T);
REPORT2("EXCESS","TOTAL") = SUM(T, REPORT2("EXCESS",T));
DISPLAY REPORT2;

PARAMETER REPORT3(*,T) PROJECT COST MEASURES;
REPORT3(I,T) $((DATA(I,"H") EQ 1) AND (BEGIN(I,T) EQ 0)) =
   DATA(I,"V")*N.L(I)*(M.L(I,T)**DATA(I,"ALPHA"))
 + DATA(I,"W")*N.L(I)*(S.L(I,T)**DATA(I,"BETA"));
REPORT3(I2,T) $((DATA2(I2,"HH") EQ 1) AND (BB(I2,T) EQ 1)) =
   CC.L(I2,T) ;
REPORT3(I,T) $((BEGIN(I,T) EQ 1) AND (DATA(I,"H") EQ 1)) =
   DATA(I,"P")*N.L(I);
REPORT3("TOTAL",T) = SUM(I $(DATA(I,"H") EQ 1), REPORT3(I,T)) +
     SUM(I2 $(DATA2(I2,"HH") EQ 1), REPORT3(I2,T));
DISPLAY REPORT3;

PARAMETER REPORT4(*,T) MAINTENANCE COST OF DRP;

REPORT4(I,T) $((DATA(I,"H") EQ 1) AND (BEGIN(I,T) EQ 0)) =
    DATA(I,"V")*N.L(I)*(M.L(I,T)**DATA(I,"ALPHA"));
REPORT4("TOTAL",T) = SUM(I $(DATA(I,"H") EQ 1), REPORT4(I,T));
DISPLAY REPORT4;

PARAMETER REPORT5(*,T) SUPPORT COST OF DRP;
REPORT5(I,T) $((DATA(I,"H") EQ 1) AND (BEGIN(I,T) EQ 0)) =
    DATA(I,"W")*N.L(I)*(S.L(I,T)**DATA(I,"BETA"));
REPORT5("TOTAL",T) = SUM(I $(DATA(I,"H") EQ 1), REPORT5(I,T));
DISPLAY REPORT5;

PARAMETER XX(I,T) NET BENEFITS OF PROJECTS;
XX(I,T) = REPORT2(I,T) - REPORT3(I,T);
DISPLAY XX;

DISPLAY COST.M;
DISPLAY N.L;
```

APPENDIX B: RATE OF RETURN OF REFERENCE PROJECT

A. INTRODUCTION

Ideally we would like to have all the reference projects to be selected with some diminishing-return projects, so that their rates of return can be used appropriately as the discount rates under the concept of the opportunity cost of rate of return. However, this was found to be not achievable in this present model. In this problem, all projects begin in year 1. When different values of discount rates are used, the reference projects selected for funding are either reference project 1 or reference projects 2 and 3. The only time when all three reference projects coexist in the funded set is when there are no other diminishing projects. This is because both the benefit and cost are linearly related to the number of systems.

To be consistent with the opportunity cost of rate of return, the rate of return of the reference project that is not selected has to be adjusted upward until it becomes the marginal rate. In this section, we show how this is done.

B. INPUT PARAMETERS

Table B1 show the input parameters. This same set of inputs is used in the main text as well.

Table B1 Input Parameters for Projects

	Exponents in Benefit Function		Unit System Price	Exponents in Cost Function		Conversion Factors			Residual Maintenance
Project	a	b	K	α	β	u	v	w	d
1	0.6	0.4	0.14	1.5	2.0	0.56	0.21	0.66	0.1
2	0.5	0.4	0.28	1.8	2.0	0.70	0.35	0.20	0.1
3	0.2	0.8	0.30	2.0	1.5	0.70	0.45	0.40	0.1

C. SUMMARY OF RESULTS

Table B2 Summary of Results

Case	1	2	3a	3b	4
r1	0.30	0.53	0.536	0.537	0.48
r2	0.30	0.30	0.30	0.30	0.28
r3	0.30	0.30	0.30	0.30	0.30
Proj 1	-	-	-	-	-
Proj 2	Y	Y	Y	-	Y
proj 3	-	-	-	-	-
Ref Proj 1	-	-	-	Y	-
Ref Proj 2	Y	Y	Y	Y	Y
Ref Proj 3	Y	Y	Y	Y	Y
Discounted Output (Z)	44.95	38.19	38.04	38.02	38.28
DPV	+2.73	+0.06	0.004	0.0	+0.10

1. CASE 1

Rate of Return of Reference Projects (0.3, 0.3, 0.3)

Total Discounted Output, Z = 44.95

Table B3.1 Output Measure (undiscounted)

	Year 2	Year 3	Year 4	Total
Project 2	24.9	25.0		49.9
Ref. Proj. 2		9.9		9.9
Ref. Proj. 3			11.4	11.4
Total	24.9	34.9	11.4	71.1

Table B3.2 Cost Measure

	Year 1	Year 2	Year 3
Project 2	15.0	12.4	11.3
(Maint. &		7.4	6.3
Sup.)		5.0	5.0
Ref. Proj. 2		7.6	
Ref. Proj. 3			8.7
Total	15.0	20.0	20.0

Table B3.3 Net Benefits for Projects

	Year 1	Year 2	Year 3	Year 4	DPV
Project 2	-15.0	12.5	13.7		2.73
Ref. Proj. 2		-7.6	9.9		0
Ref. Proj. 3			-8.7	11.4	0
Net	-15.0	4.9	14.9	11.4	2.73

Table B3.4 Shadow Prices

	Year 1	Year 2	Year 3
Shadow Prices	1.182	0.769	0.592

2. CASE 2

Rate of Return of Reference Projects (0.53, 0.3, 0.3)

Total Discounted Output, Z = 38.19

Table B4.1 Output Measure (undiscounted)

	Year 2	Year 3	Year 4	Total
Project 2 Ref. Proj. 2 Ref. Proj. 3	24.9	25.0 9.9	 11.4	49.9 9.9 11.4
Total	24.9	34.9	11.4	71.1

Table B4.2 Cost Measure

	Year 1	Year 2	Year 3
Project 2 (Maint. & Sup.)	15.0	12.4 7.4 5.0	11.3 6.3 5.0
Ref. Proj. 2 Ref. Proj. 3		7.6	 8.7
Total	15.0	20.0	20.0

Table B4.3 Net Benefits for Projects

	Year 1	Year 2	Year 3	Year 4	DPV
Project 2 Ref. Proj. 2 Ref. Proj. 3	-15.0	12.5 -7.6	13.8 9.8 -8.7	 11.4	0.063 0 0
Net	-15.0	4.9	14.9	11.4	0.063

Table B4.4 Shadow Prices

	Year 1	Year 2	Year 3
Shadow Prices	1.004	0.654	0.503

3. CASE 3A

Rate of Return of Reference Projects (0.536, 0.3, 0.3)

Total Discounted Output, Z = 38.041

Table B4.1 Output Measure (undiscounted)

	Year 2	Year 3	Year 4	Total
Project 2	24.9	25.0		49.9
Ref. Proj. 2		9.9		9.9
Ref. Proj. 3			11.4	11.4
Total	24.9	34.9	11.4	71.1

Table B4.2 Cost Measure

	Year 1	Year 2	Year 3
Project 2	15.0	12.4	11.3
(Maint. &		7.4	6.3
Sup.)		5.0	5.0
Ref. Proj. 2		7.6	
Ref. Proj. 3			8.7
Total	15.0	20.0	20.0

Table B4.3 Net Benefits for Projects

	Year 1	Year 2	Year 3	Year 4	DPV
Project 2	-15.0	12.5	13.8		0.004
Ref. Proj. 2		-7.6	9.9		0
Ref. Proj. 3			-8.7	11.4	0
Net	-15.0	4.9	14.9	11.4	0.004

Table B4.4 Shadow Prices

	Year 1	Year 2	Year 3
Shadow Prices	1.001	0.651	0.501

4. **CASE 3B**

Rate of Return of Reference Projects (0.537, 0.3, 0.3)

Total Discounted Output, Z = 38.02

Table B5.1 Output Measure (undiscounted)

	Year 2	Year 3	Year 4	Total
Ref. Proj. 1	23.1			23.1
Ref. Proj. 2		26.0		26.0
Ref. Proj. 3			26.0	26.0
Total	23.1	26.0	26.0	75.1

Table B5.2 Cost Measure

	Year 1	Year 2	Year 3
Ref. Proj. 1	15.0		
Ref. Proj. 2		20.0	
Ref. Proj. 3			20.0
Total	15.0	20.0	20.0

Table B5.3 Net Benefits for Projects

	Year 1	Year 2	Year 3	Year 4	DPV
Ref. Proj. 1	-15.0	23.1			0
Ref. Proj. 2		-20.0	26.0		0
Ref. Proj. 3			-20.0	26.0	0
Net	-15.0	3.1	6.0	26.0	0.0

Table B5.4 Shadow Prices

	Year 1	Year 2	Year 3
Shadow Prices	1.000	0.651	0.500

5. CASE 4

Rate of Return of Reference Projects (0.537, 0.28, 0.3)

Total Discounted Output, Z = 38.28

Table B6.1 Output Measure (undiscounted)

	Year 2	Year 3	Year 4	Total
Project 2	24.9	25.0		49.9
Ref. Proj. 2		9.7		9.7
Ref. Proj. 3			11.4	11.4
Total	24.9	34.7	11.4	71.0

Table B6.2 Cost Measure

	Year 1	Year 2	Year 3
Project 2	15.0	12.4	11.3
(Maint. &		7.5	6.3
Sup.)		5.0	5.0
Ref. Proj. 2		7.6	
Ref. Proj. 3			8.7
Total	15.0	20.0	20.0

Table B6.3 Net Benefits for Projects

	Year 1	Year 2	Year 3	Year 4	DPV
Project 2	-15.0	12.5	13.7		0.102
Ref. Proj. 2		-7.6	9.7		0
Ref. Proj. 3			-8.7	11.4	0
Net	-15.0	4.9	14.7	11.4	0.102

Table B6.4 Shadow Prices

	Year 1	Year 2	Year 3
Shadow Prices	1.007	0.651	0.508

APPENDIX C: NON-OPTIMAL PORTFOLIOS

A. CASE 1 (REMOVE PROJECT 2)

Rate of Return of Reference Projects (0.536, 0.3, 0.3)

Total Discounted Output, Z = 38.037

Table C1.1 Undiscounted Output of the Unfunded Projects

Unfunded Set	Year 2	Year 3	Total
Ref. Proj. 1	23.0	-	23.0
Ref. Proj. 2	-	26.0	26.0
Ref. Proj. 3	-	-	26.0
Total	23.0	26.0	75.0

Table C1.2 Cost Measure of the Unfunded Projects

Unfunded Set	Year 1	Year 2	Year 3
Ref. Proj. 1	15.0	-	-
Ref. Proj. 2	-	20.0	-
Ref. Proj. 3	-	-	20.0
Total	15.0	20.0	20.0

Table C1.3 Discounted Present Value (DPV) of the Net Benefits for the Funded Projects

Unfunded Set	Year 1	Year 2	Year 3	Year 4	DPV
Ref. Proj. 1	-15.0	23.0	-	-	0
Ref. Proj. 2	-	-20.0	26.0	-	0
Ref. Proj. 3	-	-	-20.0	26.0	0
Total	-15.0	3.0	6.0	26.0	0

B. CASE 2 (REMOVE REFERENCE PROJECT 2)

Rate of Return of Reference Projects (0.536, 0.3, 0.3)

Total Discounted Output, Z = 37.435

Table C2.1 Undiscounted Output of the Unfunded Projects

Unfunded Set	Year 2	Year 3	Year 4	Total
Project 2	31.2	25.3	-	56.6
Ref. Proj. 3	-	-	11.4	11.4
Total	31.2	25.3	11.4	68.0

Table C2.2 Cost Measure of the Unfunded Projects

Unfunded Set	Year 1	Year 2	Year 3
Project 2	15.0	20.0	11.2
(Maint. &	-	12.0	6.2
Sup.)	-	8.0	5.0
Ref. Proj. 3	-	-	8.8
Total	15.0	20.0	20.0

Table C2.3 Discounted Present Value (DPV) of the Net Benefits for the Funded Projects

Unfunded Set	Year 1	Year 2	Year 3	Year 4	DPV
Project 2	-15.0	11.2	14.1	-	-0.6
Ref. Proj. 3	-	-	-8.8	11.4	0
Total	-15.0	11.2	5.3	11.4	-0.6

C. CASE 3 (REMOVE REFERENCE PROJECT 3)

Rate of Return of Reference Projects (0.536, 0.3, 0.3)

Total Discounted Output, Z = 37.396

Table C3.1 Undiscounted Output of the Unfunded Projects

Unfunded Set	Year 2	Year 3	Total
Project 2	24.9	32.5	57.3
Ref. Proj. 2	-	9.9	9.9
Total	24.9	42.4	67.2

Table C3.2 Cost Measure of the Unfunded Projects

Unfunded Set	Year 1	Year 2	Year 3
Project 2	15.0	12.4	20.0
(Maint. &	-	7.4	11.3
Sup.)	-	5.0	8.7
Ref. Proj. 2	-	7.6	-
Total	15.0	20.0	20.0

Table C3.3 Discounted Present Value (DPV) of the Net Benefits for the Funded Projects

Unfunded Set	Year 1	Year 2	Year 3	DPV
Project 2	-15.0	12.5	12.5	-0.64
Ref. Proj. 2	-	-7.6	9.9	0
Total	-15.0	4.9	22.4	-0.64

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